# Derivata di $[\,a^{x}\,]_{x \in \mathbb{R}}$

**condizione di definibilità della funzione: a > 0**

$\text{der}(a^{x}, x)$

$\text{der}(a^{x}, x) = \lim\left(\frac{\left[\,[a^{x}]\,\right]_{x = x + \Delta x} - a^{x}}{\Delta x}, \Delta x, 0\right)$ Transform

$\text{der}(a^{x}, x) = \lim\left(\frac{a^{\Delta x + x} - a^{x}}{\Delta x}, \Delta x, 0\right)$ Simplify

$\text{der}(a^{x}, x) = \lim\left(\frac{a^{\Delta x}\, a^{x} - a^{x}}{\Delta x}, \Delta x, 0\right)$ Collect

$\text{der}(a^{x}, x) = \lim\left(\frac{a^{x}\left[a^{\Delta x} - 1\right]}{\Delta x}, \Delta x, 0\right)$ Collect

$\text{der}(a^{x}, x) = \lim\left(a^{x}\, \frac{a^{\Delta x} - 1}{\Delta x}, \Delta x, 0\right)$ Transform

$\text{der}(a^{x}, x) = \lim(a^{x}, \Delta x, 0)\, \lim\left(\frac{a^{\Delta x} - 1}{\Delta x}, \Delta x, 0\right)$ Expand

$\text{der}(a^{x}, x) = a^{x} \lim\left(\frac{a^{\Delta x} - 1}{\Delta x}, \Delta x, 0\right)$ Transform

$y = \frac{a^{\Delta x} - 1}{\Delta x}$

$L = \lim\left(\frac{a^{\Delta x} - 1}{\Delta x}, \Delta x, 0\right)$

$\text{der}(a^{x}, x) = a^{x} L$ Substitute

$z = a^{\frac{1}{y}}$

$z = a^{\frac{1}{\frac{a^{\Delta x} - 1}{\Delta x}}}$ Substitute

$z = a^{\frac{\Delta x}{a^{\Delta x} - 1}}$ Simplify

$z = a^{\Delta x \frac{1}{a^{\Delta x} - 1}}$ Transform

$z = \left(a^{\Delta x}\right)^{\frac{1}{a^{\Delta x} - 1}}$ Collect

$w = \frac{1}{a^{\Delta x} - 1}$

$a^{\Delta x} = \frac{1}{w} + 1$ Isolate

$\lim(w, \Delta x, 0)$

$\lim(w, \Delta x, 0) = \lim\left(\frac{1}{a^{\Delta x} - 1}, \Delta x, 0\right)$ Substitute

$\lim(w, \Delta x, 0) = \infty$ Transform

$z = \left(a^{\Delta x}\right)^{w}$ Substitute

$z = \left(\frac{1}{w} + 1\right)^{w}$ Substitute

△ $a^{\frac{1}{y}} = \left(\frac{1}{w}+1\right)^{w}$ **Substitute**

△ $y = \frac{1}{\log_{a}\left(\left[\frac{1}{w}+1\right]^{w}\right)}$ **Isolate**

△ $\frac{a^{\Delta x}-1}{\Delta x} = \frac{1}{\log_{a}\left(\left[\frac{1}{w}+1\right]^{w}\right)}$ **Substitute**

△ $L = \lim\left(\frac{1}{\log_{a}\left[\left\{\frac{1}{w}+1\right\}^{w}\right]}, \Delta x, 0\right)$ **Substitute**

△ $L = \frac{\lim(1, \Delta x, 0)}{\lim\left(\log_{a}\left[\left\{\frac{1}{w}+1\right\}^{w}\right], \Delta x, 0\right)}$ **Simplify**

△ $L = \frac{1}{\lim\left(\log_{a}\left[\left\{\frac{1}{w}+1\right\}^{w}\right], \Delta x, 0\right)}$ **Transform**

△ $L = \frac{1}{\log_{a}\left(\lim\left[\left\{\frac{1}{w}+1\right\}^{w}, \Delta x, 0\right]\right)}$ **Transform**

△ $L = \frac{1}{\log_{a}\left(\lim\left[\left\{\frac{1}{w}+1\right\}^{w}, w, \lim\{w, \Delta x, 0\}\right]\right)}$ **Substitute**

△ $L = \frac{1}{\log_{a}\left(\lim\left[\left\{\frac{1}{w}+1\right\}^{w}, w, \infty\right]\right)}$ **Substitute**

△ $L = \frac{1}{\log_{a}(e)}$ **Simplify**

△ $\mathrm{der}\left(a^{x}, x\right) = a^{x}\frac{1}{\log_{a}(e)}$ **Substitute**

△ $\mathrm{der}\left(a^{x}, x\right) = a^{x}\frac{1}{\frac{1}{\log_{e}(a)}}$ **Transform**

△ $\mathrm{der}\left(a^{x}, x\right) = a^{x}\log_{e}(a)$ **Simplify**

△ $\mathrm{der}\left(a^{x}, x\right) = a^{x}\ln(a)$ **Transform**